Trigonometric Identities and Equations- Questions

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

(a) Solve, for $-180° \leqslant \theta \leqslant 180°$, the equation

$$5\sin 2\theta = 9\tan\theta$$

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(b) Deduce the smallest positive solution to the equation

$$5\sin(2x - 50°) = 9\tan(x - 25°)$$

(2)

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2.

The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)° \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

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3.

(a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant x < 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

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4.

(a) Show that the equation

$$4\cos\theta - 1 = 2\sin\theta\tan\theta$$

can be written in the form

$$6\cos^2\theta - \cos\theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 90°$

$$4\cos 3x - 1 = 2\sin 3x\tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

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5.

8. (*a*) Show that the equation

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2$$

(3)

(*b*) Hence solve, for $0 \leqslant x < 360°$,

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

giving your answers to 2 decimal places.

(5)

May 2016 Mathematics Advanced Paper 1: Pure Mathematics 2

6.

6. (i) Solve, for $-\pi < \theta \leq \pi$,

$$1 - 2\cos\left(\theta - \frac{\pi}{5}\right) = 0,$$

giving your answers in terms of π.

(3)

(ii) Solve, for $0 \leq x < 360°$,

$$4\cos^2 x + 7\sin x - 2 = 0,$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

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7.

8. (i) Solve, for $0 \leq \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3} \cos 3\theta = 0,$$

giving your answers in terms of π.

(3)

(ii) Given that

$$4 \sin^2 x + \cos x = 4 - k, \qquad 0 \leq k \leq 3,$$

(*a*) find $\cos x$ in terms of k.

(3)

(*b*) When $k = 3$, find the values of x in the range $0 \leq x < 360°$.

(3)

May 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

8.

8. (i) Solve, for $-180° \leq x < 180°$,

$$\tan(x - 40°) = 1.5,$$

giving your answers to 1 decimal place.

(3)

(ii) (*a*) Show that the equation

$$\sin \theta \tan \theta = 3 \text{ co s}\theta + 2$$

can be written in the form

$$4 \cos^2\theta + 2 \cos \theta - 1 = 0.$$

(3)

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$\sin \theta \tan \theta = 3 \cos \theta + 2,$$

showing each stage of your working.

(5)

Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

9.

4. Solve, for $0 \leq x < 180°$,

$$\cos(3x - 10°) = -0.4,$$

giving your answers to 1 decimal place. You should show each step in your working.

(7)

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10.

6. (*a*) Show that the equation

$$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5\cos 2x)\sin 2x = 0.$$

(2)

(*b*) Hence solve, for $0 \leq x \leq 180°$,

$$\tan 2x = 5 \sin 2x,$$

giving your answers to 1 decimal place where appropriate.
You must show clearly how you obtained your answers.

(5)

Jan 2012 Mathematics Advanced Paper 1: Pure Mathematics 2

11.

9. (i) Find the solutions of the equation $\sin(3x - 15°) = \frac{1}{2}$, for which $0 \leq x \leq 180°$.

(6)

(ii)

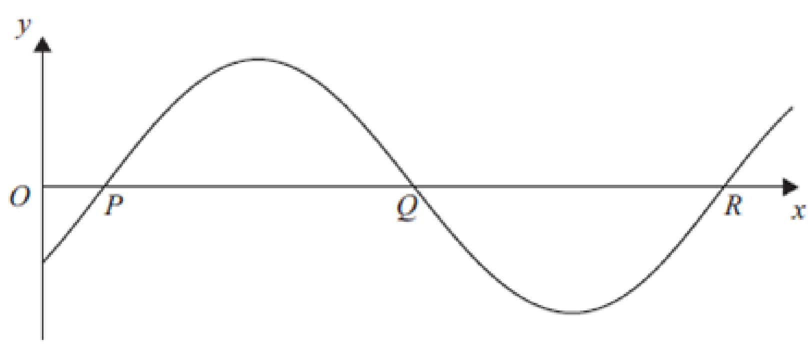

Figure 4

Figure 4 shows part of the curve with equation

$$y = \sin(ax - b), \text{ where } a > 0, \quad 0 < b < \pi.$$

The curve cuts the x-axis at the points P, Q and R as shown.

Given that the coordinates of P, Q and R are $\left(\frac{\pi}{10}, 0\right)$, $\left(\frac{3\pi}{5}, 0\right)$ and $\left(\frac{11\pi}{10}, 0\right)$ respectively, find the values of a and b.

(4)

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12.

7. (*a*) Solve for $0 \leq x < 360°$, giving your answers in degrees to 1 decimal place,

$$3 \sin (x + 45°) = 2.$$

(4)

(*b*) Find, for $0 \leq x < 2\pi$, all the solutions of

$$2 \sin^2 x + 2 = 7\cos x,$$

giving your answers in radians.

You must show clearly how you obtained your answers.

(6)

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 2

13.

7. (*a*) Show that the equation

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

can be written in the form

$$4 \sin^2 x + 7 \sin x + 3 = 0.$$

(2)

(*b*) Hence solve, for $0 \leq x < 360°$,

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

Jun 2010 Mathematics Advanced Paper 1: Pure Mathematics 2

14.

5. (*a*) Given that $5 \sin \theta = 2 \cos \theta$, find the value of $\tan \theta$.

(1)

(*b*) Solve, for $0 \leq x < 360°$,

$$5 \sin 2x = 2 \cos 2x,$$

giving your answers to 1 decimal place.

(5)

Jan 2010 Mathematics Advanced Paper 1: Pure Mathematics 2

15.

2. (*a*) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0.$$

(2)

(*b*) Solve, for $0 \leq x < 360°$,

$$2 \sin^2 x + 5 \sin x - 3 = 0.$$

(4)

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16.

3. Given that

$$2 \cos (x + 50)° = \sin (x + 40)°.$$

(*a*) Show, without using a calculator, that

$$\tan x° = \frac{1}{3} \tan 40°.$$

(4)

(*b*) Hence solve, for $0 \leq \theta < 360$,

$$2 \cos (2\theta + 50)° = \sin (2\theta + 40)°,$$

giving your answers to 1 decimal place.

(4)

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 3

17.

3. Find all the solutions of

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0 \leq \theta < 360°$.

(6)

June 2010 Mathematics Advanced Paper 1: Pure Mathematics 3

18.

1. (*a*) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta.$$

(2)

(*b*) Hence find, for $-180° \leq \theta < 180°$, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1.$$

Give your answers to 1 decimal place.

(3)